Mathematics Olympiad Worksheet: Divisibility Rules

- **Divisibility by 2, 5, 10:** Depends only on the last digit ($2 \to$ even, $5 \to 0$ or 5, $10 \to 0$).
- **Divisibility by 3, 9:** Sum of all digits must be divisible by 3 or 9, respectively.
- **Divisibility by 4, 25:** The number formed by the last two digits must be divisible by 4 or 25.
- **Composite Rules (6, 15, 36, 45):** Test coprimes ($6 \to 2$ and 3; $15 \to 3$ and 5; $36 \to 4$ and 9; $45 \to 5$ and 9).

Part 1: Problems from Textbook

1. Fill in the missing digits represented by asterisks ($*$):

a) Find a so that $2a45$ is divisible by 9.

b) Find digits a and b so that $29a45b$ is divisible by 45.

c) Find digits a and b so that $72a4b$ is divisible by 45.

d) Find digits a and b so that $1a456b$ is divisible by 36.

2. Prove that the following numbers are composite: 111, 221, 1111, 100001, 107107, 1234554321.

3. Which is the largest 3-digit number that is divisible by both 2 and 9?

a) 900 **b)** 981 **c)** 990 **d)** 918 **e)** 999

4. How many 2-digit numbers exist that are multiples of 9?

5. At the end of a lesson, students handed in their test notebooks and exercise notebooks (a total of 47 notebooks). Did every student hand in both notebooks? Explain your reasoning.

6. Replace the asterisk ($*$) with a digit so that the number becomes divisible by 9:

1) $1*00$ **2)** $25*8$ **3)** $4*1$ **4)** $*888$

7. What digit can replace the asterisk ($*$) in each case?

1) $*54312$ is divisible by 3

2) $4532*$ is divisible by 5

3) $32*25$ is divisible by 5

4) $*3260$ is divisible by 5

5) $423*0$ is divisible by 10

6) $*2310$ is divisible by 10

8. What is the smallest positive integer that must be added to the following numbers to make them divisible by 9?

1) 1275 **2)** 33333 **3)** 10203040 **4)** 1919191919

9. Which of the following numbers is divisible by 6?

a) 273 **b)** 412 **c)** 1002 **d)** 3004

10. Calculate: $400 \div (25 - 75) - 200 \div (13 - 33)$

a) 2 **b)** 18 **c)** -18 **d)** -2

11. What is the smallest natural number that must be added to 2011 so that the result is divisible by 9?

a) 8 **b)** 2 **c)** 5 **d)** 3

12. Which of the following numbers is divisible by 15?

a) 5005 **b)** 4005 **c)** 1015 **d)** 2015

13. Using the digits $2, 5$, and 9 without repetition, which type of 3-digit number CANNOT be formed?

a) A multiple of 5

b) An even number

c) A multiple of 3

d) An odd number

14. The smallest natural number written using only the digit 2 that is divisible by 3 is greater than the smallest 3-digit number by how much?

a) 122 **b)** 112 **c)** 123 **d)** 827

15. What digit should replace the asterisk in $123*5$ so that the resulting 5-digit number is divisible by 45?

a) 9 **b)** 8 **c)** 3 **d)** 7

16. The number $*35*$ (where both asterisks represent the exact same digit) is divisible by 15. What digit is represented by the asterisk?

17. Find the smallest natural number of the form $\overline{123X43Y}$ that is divisible by 3.

18. Find all 4-digit numbers $\overline{2a3b}$ that are divisible by 12.

19. What is the remainder when the 100-digit number $111\ldots11$ (containing one hundred 1s) is divided by 9?

Part 2: Basic Divisibility & Word Problems

1. A furniture factory makes two types of chairs: 3-legged and 4-legged. In stock, they have a total of 786,484 chair legs. All legs must be used.

1) Can they manufacture the exact same number of 3-legged and 4-legged chairs?

2) What is the maximum total number of chairs that can be made?

3) What is the minimum total number of chairs that can be made?

4) If one leg costs between 50 tetri and 80 tetri, and the total cost of all legs is a whole number of Lari, what is the total cost of all legs?

2. Find the smallest natural number that must be added to:

1) 4766 to make it divisible by 3

2) 47831 to make it divisible by 9

3) 87898 to make it divisible by 3

4) 35987 to make it divisible by 9

3. Solve the following pricing word problems:

1) 6 identical pencils cost more than 73 tetri and less than 81 tetri. What was the exact price of one pencil in tetri?

2) 37 identical erasers cost more than 12 Lari and less than 12.5 Lari. What was the price of one eraser in tetri?

3) Students raised money for a trip (more than 350 Lari and less than 370 Lari). Each student paid 17 Lari. How many students went on the trip?

4) Employees raised money (more than 335 Lari and less than 355 Lari). Each paid 15 Lari. How many employees are there?

Part 3: Remainder Theory Problems

4. What is the remainder when 4001 is divided by the sum of its digits?

a) 1 **b)** 2 **c)** 3 **d)** 4 **e)** 5

5. What is the smallest 3-digit number whose sum of digits equals 22?

a) 589 **b)** 499 **c)** 225 **d)** 522 **e)** 679

6. If 5 is added to a natural number N, the result is divisible by 9. What is the remainder when N is divided by 3?

a) 5 **b)** 0 **c)** 1 **d)** 2 **e)** 4

7. When a natural number n is divided by 12, the remainder is r. What is the maximum possible value of r?

8. When 50 is divided by m, the remainder is 8.

1) What is the smallest possible value of m?

2) What is the largest possible value of m?

9. What is the smallest two-digit natural number that leaves a remainder of 1 when divided by both 6 and 8?

10. Find the two smallest natural numbers that leave a remainder of 2 when divided by 4, and a remainder of 3 when divided by 5.

11. Find the smallest natural number that leaves a remainder of 6 when divided by 7, and a remainder of 8 when divided by 9.

12. Find the smallest natural number that is divisible by 7, but leaves a remainder of 1 when divided by 3, 4, and 5.

13. When a natural number is divided by 12, the remainder is 5. What remainder is obtained when the same number is divided by 6?